$$\frac{d^2T}{dt^2} + k^2 C_1^{\,2} T = 0 \tag{23}$$

where k^2 is the constant of separation to be determined. Equation (22) is Bessel's equation and its solution is

$$R(r) = B_1\, j_1\,(kr) + B_2 y_1(kr) \tag{24}$$

where $j_1(kr)$ and $y_1(kr)$ the spherical Bessel functions of the first and second kind of the first order [11]. Since $R(r)$ is finite for $r = 0$, B_2 must be zero. Combining equation (24) and the boundary condition of equation (19), we obtain a transcendental equation for k, the constant of separation

$$\tan(ka) = (ka)/[1 - C_1^{\,2}(ka)^2/(4C_2^{\,2})] \tag{25}$$

where $C_2 = (\mu/\rho)^{1/2}$ is the velocity of shear wave propagation. The solution of equation (25) is an infinite sequence of eigenvalues, k_m; each corresponds to a characteristic mode of vibration of the spherical head. Moreover, since equation (23) is harmonic in time, a general solution for $u_t(r,t)$ may be written as

$$u_t(r,t) = \sum_{m=0}^{\infty} A_m j_1(k_m r)\,\mathrm{Cos}\,\omega_m t \tag{26}$$

where

$$\omega_m = k_m C_1 \tag{27}$$

and ω_m is the angular frequency of vibration of the sphere. We evaluate the constants A_m by using the initial conditions in equation (14) to obtain

$$A_m = -u_\circ \left\{ \frac{a}{N\pi} \int_\circ^a r^2 j_1(k_m r) j_1\left(\frac{N\pi r}{a}\right) dr \pm \frac{4\mu}{3\lambda+2\mu}\left(\frac{1}{N^2\pi^2}\right) \int_\circ^a r^3 j_1(k_m r) dr \right\}$$

$$\Big/ \left\{ \int_\circ^a r^2 [j_1(k_m r)]^2\, dr \right\}, \qquad N = \begin{cases} 1,3,5\ldots \\ 0,2,4\ldots \end{cases} \tag{28}$$

The integrals in equation (28) may be evaluated [17] to give

$$A_m = \mp u_\circ a \left(\frac{1}{N\pi}\right)^2 \left\{ \frac{2}{[j_1(k_m a)]^2 - j_\circ(k_m a) j_2(k_m a)} \right\} \left\{ \frac{4\mu}{3\lambda+2\mu}\, \frac{1}{k_m a}\, j_2(k_m a) \right.$$

$$\left. - k_m a j_\circ(k_m a)\, \frac{1}{(k_m a)^2 - (N\pi)^2} \right\}, \qquad N = \begin{cases} 1,3,5\ldots \\ 0,2,4\ldots \end{cases} \tag{29}$$

where $j_2(k_m a)$ is the spherical Bessel function of the first kind and second order. The displacement response of the sphere to a step input of microwave energy is now given by introducing equation (29) into equation (26) and then combining equation (20) and (26) in equation (15). We have