The initial conditions are

$$u(r,o) = \frac{\partial u(r,o)}{\partial r} = o \tag{14}$$

Our approach in the following derivations is first to obtain a solution for the case of step application of microwave energy at some instant $t = o$ and then to extend the solution to a square pulse using Duhamel's theorem [16].

Step Function Excitation

If we write the displacement $u(r,t)$ as

$$u(r,t) = u_s(r) + u_t(r,t) \tag{15}$$

and substitute equation (15) into equation (8), the equation of motion becomes two differential equations: a stationary one and a time-varying one. Thus,

$$\frac{d^2u_s(r)}{dr^2} + \frac{2}{r}\frac{du_s(r)}{dr} - \frac{2}{r^2}u_s(r) = u_oF_r(r) \tag{16}$$

$$\frac{\partial^2u_t(r,t)}{\partial r^2} + \frac{2}{r}\frac{\partial u_t(r,t)}{\partial r} - \frac{2}{r^2}u_t(r,t) = \frac{1}{C_1^2}\frac{\partial^2u_t(r,t)}{\partial t^2} \tag{17}$$

The corresponding boundary conditions at $r = a$ are

$$(\lambda+2\mu)\frac{du_s}{dr} + 2\lambda\frac{u_s}{r} = 0 \tag{18}$$

and

$$(\lambda+2\mu)\frac{\partial u_t}{\partial r} + 2\lambda\frac{u_t}{r} = 0 \tag{19}$$

The solution of equation (16), using the boundary condition of equation (18), is given by

$$u_s(r) = u_o\left[\frac{a}{N\pi}j_1\left(\frac{N\pi r}{a}\right) \pm \frac{4\mu}{3\lambda+2\mu}\frac{\gamma}{N^2\pi^2}\right], \quad N = \begin{cases}1,3,5\ldots\\0,2,4\ldots\end{cases} \tag{20}$$

where $j_1\left(\frac{N\pi r}{a}\right)$ is the spherical Bessel function.

Now we let

$$u_t(r,t) = R(r)\,T(t) \tag{21}$$

and use the method of separation of variables to solve equation (17) for the time-varying component. Inserting equation (21) into equation (17) yields the two ordinary differential equations:

$$\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + \left(k^2 - \frac{2}{r^2}\right)R = 0 \tag{22}$$