$$v(r,t) = \frac{I_o}{\rho c_h} \frac{\sin \frac{N\pi r}{a}}{\frac{N\pi r}{a}} t \quad (6)$$

where ρ and c_h are the density and specific heat of brain material, respectively, and $\rho c_h = K/\kappa$.

In biological materials, the stress-wave development times are short compared with temperature equilibrium times. The temperature decay for $t > t_o$ is therefore a slowly varying function of time and becomes significant only for times greater than milliseconds. We may thus assume for $t > t_o$ that

$$v(r,t) = \frac{I_o}{\rho c_h} \frac{\sin \frac{N\pi r}{a}}{\frac{N\pi r}{a}} t_o \quad (7)$$

where t_o is duration of microwave application (pulse width).

ACOUSTIC WAVE GENERATION

We now consider the spherical head with homogeneous brain material as a linear, elastic medium. The sphere is assumed to be stress-free at its surface. The equation of motion in spherical coordinates [15] is then given by

$$\frac{\partial^2 u}{\partial u^2} + \frac{2}{r}\frac{\partial u}{\partial r} - \frac{2}{r^2} u - \frac{1}{C_1^2}\frac{\partial^2 u}{\partial t^2} = \frac{\gamma}{\lambda+2\mu}\frac{\partial v}{\partial r} \quad (8)$$

where u is the displacement of brain material, $C_1 = [(\lambda+2\mu)/\rho]^{1/2}$ is the velocity of bulk acoustic wave propagation, $\gamma = \alpha(\lambda+2/3\mu)$, α is the linear coefficient of thermal expansion, and λ and μ are Lame's constants. The right-hand side of equation (8) is the change in temperature which gives rise to the displacement. We first write

$$\frac{\gamma}{\lambda+2\mu}\frac{\partial v}{\partial r} = u_o F_r(r) F_t(t). \quad (9)$$

Hence

$$u_o = \frac{I_o}{\rho c_h} \frac{\gamma}{\lambda+2\mu} \quad (10)$$

and

$$F_r(r) = \frac{d}{dr}[\sin(\frac{N\pi r}{a})/(\frac{N\pi r}{a})] \quad (11)$$

From equations (6) and (7), we have

$$F_t(t) = \begin{cases} t, & o \le t \le t_o \\ t_o, & t \ge t_o \end{cases} \quad (12)$$

If the surface of the sphere is stress-free, then the boundary condition at $r = a$ is

$$(\lambda+2\mu)\frac{\partial u}{\partial r} + 2\lambda \frac{u}{r} = \gamma v \quad (13)$$