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Rank Reduction of Ill-Conditioned Matrices in Waveguide Junction Problems

DOUGLAS N. ZUCKERMAN, MEMBER, IEEE, AND PAUL DIAMENT, MEMBER, IEEE

***Abstract*—A new low-rank spectral expansion technique for solving the ordinarily intractable matrix equations obtained from waveguide field equivalence theorem decompositions is described. The method facilitates the analysis of waveguide discontinuity problems that resist ordinary methods of solution. The technique is illustrated for the problem of scattering at a slant interface in a rectangular waveguide.**

I. Introduction

THE integral equations, and the corresponding matrix equations, that represent scattering at a waveguide discontinuity often exhibit ill-conditioned behavior. This results in computational difficulties as inversion of such matrices is inaccurate for even large-order truncated versions of the matrix. It is shown here, however, that it may be possible to take advantage of the often relatively low effective rank of the ill-conditioned portion of the matrix to overcome such difficulties.

In the following, a typical problem, that of scattering at a waveguide discontinuity, is solved by developing equations that are exact but ill conditioned. First, field equivalence theorems are used to reduce the structure to two uniformly filled waveguides with equivalent electric and magnetic current sheets at the discontinuity surface. Integral equations for the current sheets are then derived, using the null field condition in the two simpler waveguide structures. By writing series expansions for the current sheets, the integral equations are reduced to a system of linear algebraic equations for the current expansion coefficients. These exact equations are asymptotically ill conditioned. By a low rank spectral decomposition of the matrix representing the ill-conditioned portion of the equations, it is possible to solve for the currents without

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D. N. Zuckerman is with Bell Laboratories, Holmdel, NJ 07733.
P. Diament is with the Department of Electrical Engineering and Computer Science, Columbia University, New York, NY 10027.