direction only. The right-hand side of (8) is the change in temperature which gives rise to the displacement. We first write

$$\frac{\beta}{\lambda + 2\mu}\frac{\partial v}{\partial r} = u_0 F_r(r) F_t(t). \tag{9}$$

Hence

$$u_0 = \frac{I_0}{\rho c_h}\frac{\beta}{\lambda + 2\mu} \tag{10}$$

and

$$F_r(r) = \frac{d}{dr}\left[\sin\left(\frac{N\pi r}{a}\right)\Big/\left(\frac{N\pi r}{a}\right)\right]. \tag{11}$$

From (6) and (7), we have

$$F_t(t) = \begin{cases} t, & 0 \le t \le t_0 \\ t_0, & t \ge t_0. \end{cases} \tag{12}$$

If the surface of the sphere is stress free, then the boundary condition at $r = a$ is

$$(\lambda + 2\mu)\frac{\partial u}{\partial r} + 2\lambda\frac{u}{r} = \beta v = 0. \tag{13}$$

The initial conditions are

$$u(r,0) = \frac{\partial u(r,0)}{\partial t} = 0. \tag{14}$$

Our approach in the following derivations is first to obtain a solution for the case of step of microwave energy, $F_t(t) = 1$, at some instant $t = 0$ and then to extend the solution to a rectangular pulse using Duhamel's theorem [23].

1) Unit Step: If we write the displacement $u(r,t)$ as

$$u(r,t) = u_s(r) + u_t(r,t) \tag{15}$$

and substitute (15) into (8), the equation of motion becomes two differential equations: a stationary one and a time-varying one. Thus

$$\frac{d^2 u_s(r)}{dr^2} + \frac{2}{r}\frac{du_s(r)}{dr} - \frac{2}{r^2}u_s(r) = u_0 F_r(r) \tag{16}$$

and

$$\frac{\partial^2 u_t(r,t)}{\partial r^2} + \frac{2}{r}\frac{\partial u_t(r,t)}{\partial r} - \frac{2}{r^2}u_t(r,t) = \frac{1}{c_1{}^2}\frac{\partial^2 u_t(r,t)}{\partial t^2}. \tag{17}$$

The corresponding boundary conditions at $r = a$ are

$$(\lambda + 2\mu)\, du_s/dr + 2\lambda u_s/r = 0 \tag{18}$$

and

$$(\lambda + 2\mu)\, \partial u_t/\partial r + 2\lambda u_t/r = 0. \tag{19}$$

To obtain $u_s(r)$, we assume a solution of the form

$$u_s(r) = u_p(r) + D_1/r^2 + D_2 r \tag{20}$$

where $u_p(r)$ is a particular solution of (16). We now rewrite the left-hand side of (16) as follows:

$$\frac{d}{dr}\left[\frac{1}{r^2}\frac{d(r^2 u_p)}{dr}\right] = u_0 F_r(r). \tag{21}$$

We then integrate (21) from 0 to r to get the expression

$$u_p(r) = u_0\left(\frac{a}{N\pi}\right) j_1\left(\frac{N\pi r}{a}\right). \tag{22}$$

Since $u_s(r)$ must remain finite as $r \to 0$, D_1 reduces immediately to zero. The coefficient D_2 is obtained by applying the boundary condition of (18), and it is

$$D_2 = \pm u_0\left(\frac{1}{N^2\pi^2}\right)\frac{4\mu}{3\lambda + 2\mu}, \qquad N = \begin{cases} 1,3,5\cdots \\ 2,4,6\cdots \end{cases}. \tag{23}$$

The solution of (16) is therefore given by

$$u_s(r) = u_0\left[\frac{a}{N\pi} j_1\left(\frac{N\pi r}{a}\right) \pm \frac{4\mu}{3\lambda + 2\mu}\frac{r}{N^2\pi^2}\right], \qquad N = \begin{cases} 1,3,5\cdots \\ 2,4,6\cdots \end{cases} \tag{24}$$

where $j_1(N\pi r/a)$ is the spherical Bessel function of the first kind and first order.

Now we let

$$u_t(r,t) = R(r)T(t) \tag{25}$$

and use the method of separation of variables to solve (17) for the time-varying component. Inserting (25) into (17) yields the two ordinary differential equations

$$\frac{d^2 R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + \left(k^2 - \frac{2}{r^2}\right)R = 0 \tag{26}$$

$$\frac{d^2 T}{dt^2} + k^2 c_1{}^2 T = 0 \tag{27}$$

where k is the constant of separation to be determined. Equation (26) is Bessel's equation and its solution is [17]

$$R(r) = B_1 j_1(kr) + B_2 y_1(kr) \tag{28}$$

where $j_1(kr)$ and $y_1(kr)$ are the spherical Bessel functions of the first and second kind of the first order. Since $R(r)$ is finite at $r = 0$, B_2 must be zero. Combining (28) and the boundary condition of (19), we obtain a transcendental equation for k, the constant of separation,

$$\tan(ka) = (ka)/[1 - (\lambda + 2\mu)(ka)^2/(4\mu)]. \tag{29}$$

The solution of (29) is an infinite sequence of eigenvalues k_m; each corresponds to a characteristic mode of vibration of the spherical head. It can be shown that, using the values for brain matter given in Table I, $k_m a = m\pi$, $m = 1,2,3\cdots$ to within an accuracy of 10^{-7}. Moreover, since (27) is harmonic in time, a general solution for $u_t(r,t)$ may be written as

$$u_t(r,t) = \sum_{m=1}^{\infty} A_m j_1(k_m r)\cos\omega_m t \tag{30}$$

where

$$\omega_m = k_m c_1 = m\pi c_1/a \tag{31}$$

and ω_m is the angular frequency of vibration of the sphere. Note that the frequency of vibration is independent of the absorbed energy pattern. It is only a function of the spherical head size and the elastic properties of the medium.