



Fig. 3. The approximated absorbed energy distribution.

tions, respectively, and  $\bar{M}$  and  $\bar{N}$  are vector spherical wave functions. A derivation of (2) may be found in [17]. The detailed expressions are also given in [18].

For humans exposed to 918-MHz radiations and small animals such as cats exposed to 2450 MHz, the absorbed energy distributions inside the head computed from (1) and (2) show absorption peaks in the center of the head [19], [20]. Plots of the absorbed energy distribution along the three rectangular coordinate axes of a 7.0-cm-radius spherical head exposed to 918 MHz and a 3.0-cm-radius spherical head exposed to 2450-MHz plane waves are shown in Figs. 1 and 2. The plane wave impinges from the negative  $z$  direction and is polarized in the  $x$  direction. Note that in both cases the absorbed energy along the three coordinate axes exhibits characteristic oscillations along the outer portion of the spherical head and reaches a maximum near the center.

Although the detailed absorption along the three axes is not the same, we will assume a spherically symmetric absorption pattern and approximate the absorbed energy distribution inside the head by the spherically symmetric function

$$W(r,t) = I_0 \sin\left(\frac{N\pi r}{a}\right) \Big/ \left(\frac{N\pi r}{a}\right) \quad (3)$$

where  $I_0$  is the peak absorbed energy per unit volume,  $r$  is the radial variable, and  $a$  is the radius of the spherical head. The parameter  $N$  specifies the number of oscillations in the approximated spatial dependence of the absorbed energy. Fig. 3 shows the approximated energy absorption pattern for  $N = 6$  and is particularly suited for the cases shown in Figs. 1 and 2. For some frequencies and sphere sizes, the integer  $N$  may be changed to account for the difference in absorption patterns. For instance,  $N = 3$  may be chosen to

approximate the absorption pattern inside a 5-cm-radius spherical head exposed to 918-MHz radiation [20]. For other frequencies and sphere sizes, a different function will be required to describe the absorbed energy distribution.

### B. Temperature Rise

We take advantage of the symmetry of the absorbed energy pattern by expressing the heat conduction equation as a function of  $r$  alone [21]. That is,

$$\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial v}{\partial r} - \frac{1}{\kappa} \frac{\partial v}{\partial t} = \frac{-W(r,t)}{K} \quad (4)$$

where  $v$  is temperature,  $\kappa$  and  $K$  are, respectively, the thermal diffusivity and conductivity of brain matter, and  $W$  is the heat production rate, which is the same as the absorbed microwave energy pattern and is assumed for the moment to be constant over time.

Because microwave absorption occurs in a very short time interval, there will be little chance for heat conduction to take place. We may therefore neglect the spatial derivatives in (4) such that

$$\frac{1}{\kappa} \frac{dv}{dt} = \frac{W}{K} \quad (5)$$

Equation (5) may be integrated, directly, to give the change in temperature by setting the initial temperatures equal to zero. Thus

$$v(r,t) = \frac{I_0}{\rho c_h} \frac{\sin(N\pi r/a)}{N\pi r/a} t \quad (6)$$

where  $\rho$  and  $c_h$  are the density and specific heat of brain matter, respectively, and  $\rho c_h = K/\kappa$ .

In biological materials, the stress-wave development times are short compared with temperature equilibrium times. The temperature decay is therefore a slowly varying function of time and becomes significant only for times greater than milliseconds. We may thus assume for a square pulse of microwave energy, immediately after termination of radiation, that

$$v(r,t) = \frac{I_0}{\rho c_h} \frac{\sin(N\pi r/a)}{N\pi r/a} t_0 \quad (7)$$

where  $t_0$  is the pulsewidth.

### C. Sound Generation

We now consider the spherical head with homogeneous brain matter as a linear, elastic medium without viscous damping. The thermoelastic equation of motion in spherical coordinates [22] is then given by

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r} \frac{\partial u}{\partial r} - \frac{2}{r^2} u - \frac{1}{c_1^2} \frac{\partial^2 u}{\partial t^2} = \frac{\beta}{\lambda + 2\mu} \frac{\partial v}{\partial r} \quad (8)$$

where  $u$  is the displacement of brain matter,  $c_1 = [(\lambda + 2\mu)/\rho]^{1/2}$  is the velocity of bulk acoustic wave propagation,  $\beta = \alpha(3\lambda + 2\mu)$ ,  $\alpha$  is the coefficient of linear thermal expansion, and  $\lambda$  and  $\mu$  are Lamé's constants. It should be noted that the curl of  $u$  equals zero since  $u$  is in the radial