TABLE I
THERMOELASTIC PROPERTIES OF BRAIN MATTER [14]

Specific heat, c_h	0.88 cal/gm-°C
density, ρ	1.05 gm/cm^3
coefficient of linear thermal expansion, α	4.1×10^{-5}/°C
Lame's constant, λ	2.24×10^{10} dyne/cm^2
Lame's constant, μ	10.52×10^3 dyne/cm^2
Bulk velocity of propagation, c_1	1.460×10^5 cm/sec

such that

$$u_0 = (I_0/\rho c_h)[\beta/(\lambda + 2\mu)] \tag{8}$$

and

$$F_r(r) = (d/dr)[\sin (N\pi r/a)/(N\pi r/a)] \tag{9}$$

also

$$F(t) = \begin{cases} t, & 0 < t < t_0 \\ t_0, & t > t_0. \end{cases} \tag{10}$$

For a constrained surface, the boundary condition at the surface of the sphere is expressed by

$$u(a,t) = 0. \tag{11}$$

The initial conditions are

$$u(r,0) = \partial u(r,0)/\partial t = 0. \tag{12}$$

Following a technique used previously [14], we will first solve (6) for the case of $F_t(t) = 1$ and then extend the solution to a rectangular pulse using Duhamel's principle.

Solution for $F_t(t) = 1$

We first write the displacement u as

$$u(r,t) = u_s(r) + u_t(r,t) \tag{13}$$

and substitute (13) into (6) to obtain

$$(d^2u_s/dr^2) + (2/r)(du_s/dr) - (2/r^2)u_s = u_0 F_r(r) \tag{14}$$

and

$$(\partial^2 u_t/\partial r^2) + (2/r)(\partial u_t/\partial r) - (2/r^2)u_t = (1/c_1^2)(\partial^2 u_t/\partial t^2). \tag{15}$$

The corresponding boundary conditions are

$$u_s(a) = 0 \tag{16}$$

and

$$u_t(a,t) = 0. \tag{17}$$

To facilitate the solution of (14), we let

$$u_s = u_p + Br \tag{18}$$

where u_p is a particular solution of (14) and is obtained by integrating (14) from 0 to r. Thus

$$u_p = u_0(a/N\pi)j_1(N\pi r/a) \tag{19}$$

where j_1 is the spherical Bessel function of the first kind. The coefficient B is evaluated by applying the boundary condition given in (16). The solution to (14) is therefore

$$u_s = (u_0/N\pi)[aj_1(N\pi r/a) \mp (r/N\pi)], \quad N = \begin{cases} 1, 3, 5, \cdots \\ 2, 4, 6, \cdots. \end{cases} \tag{20}$$

Next we let

$$u_t = R(r)T(t) \tag{21}$$

and solve (15) using the method of separation of variables. Substituting (21) into (15) we have

$$(d^2R/dr^2) + (2/r)(dR/dr) + (k^2 - 2/r^2)R = 0 \tag{22}$$

and

$$(d^2T/dt^2) + k^2c_1^2 T = 0 \tag{23}$$

where k is the yet undetermined constant of separation. The solution of (22) is a set of spherical Bessel functions j_1 and y_1 or

$$R = B_1 j_1(kr) + B_2 y_1(kr). \tag{24}$$

Since R is finite at $r = 0$, B_2 must be zero. Substituting (24) into the boundary condition of (17) we get an equation for the separation constant k. Thus

$$j_1(ka) = 0. \tag{25}$$

We may denote the zeros of j_1 by $k_m a$, $m = 1, 2, 3, \cdots$. The solution to (23) is clearly harmonic in time. We may write the general solution to (15) as

$$u_t = \sum_{m=1}^{\infty} A_m j_1(k_m r) \cos \omega_m t \tag{26}$$

where A_m is yet to be determined and $\omega_m = k_m c_1$ or

$$f_m = k_m c_1/2\pi. \tag{27}$$

Since f_m represents the frequency of vibration of the spherical head, there are, therefore, an infinite number of modes of vibration of the spherical head irradiated with appropriate pulse-modulated microwave energy.

To evaluate A_m, we need the initial condition $u(r,0) = 0$ and the orthogonality relations given in [14]. Thus

$$A_m = \pm 2u_0 a(1/N\pi)^2 \cdot \frac{(1/k_m a)j_2(k_m a) \pm k_m a j_0(k_m a)/[(k_m a)^2 - (N\pi)^2]}{[j_1(k_m a)]^2 - j_0(k_m a)j_2(k_m a)}, \quad N = \begin{cases} 1, 3, 5, \cdots \\ 2, 4, 6, \cdots. \end{cases} \tag{28}$$

By substituting (20) and (26) into (13), we have

$$u = u_0 D + \sum_{m=1}^{\infty} A_m j_1(k_m r) \cos \omega_m t \tag{29}$$

where

$$D = (1/N\pi)[aj_1(N\pi r/a) \mp (r/N\pi)], \quad N = \begin{cases} 1, 3, 5, \cdots \\ 2, 4, 6, \cdots. \end{cases} \tag{30}$$